

AD-751 337

ON SURFACE ELECTROMAGNETIC WAVES IN  
SYSTEMS WITH NONUNIFORM IMPEDANCE

V. I. Talanov

Morris T. Friedman, Incorporated

Prepared for:

Air Force Cambridge Research Laboratory

September 1959

DISTRIBUTED BY:

**NTIS**

National Technical Information Service  
U. S. DEPARTMENT OF COMMERCE  
5285 Port Royal Road, Springfield Va. 22151

lassified

Security Classification

AD-757 337

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Morris D. Friedman, Inc 1383 A Washington Street West Newton 75, Massachusetts	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP
--	---

3. REPORT TITLE  
ON SURFACE ELECTROMAGNETIC WAVES IN SYSTEMS WITH NONUNIFORM

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Scientific Interim

5. AUTHOR(S) (First name, middle initial, last name)

V.I. Talanov

6. REPORT DATE  
September 1959

7a. TOTAL NO. OF PAGES  
3

7b. NO. OF REFS  
3

8a. CONTRACT OR GRANT NO.  
AF19(604)-5969

b. PROJECT, TASK, WORK UNIT NOS.  
4600-07-01

c. DOD ELEMENT

N/A

d. DOD SUBELEMENT

N/A

9a. ORIGINATOR'S REPORT NUMBER(S)

(Translation)  
Scientific Report No. 2

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

AFCRC-TN-59-768 ✓

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited

11. SUPPLEMENTARY NOTES

TECH, OTHER

12. SPONSORING MILITARY ACTIVITY

Air Force Cambridge Research  
Laboratories (CRR)  
L. G. Hanscom Field  
Bedford, Massachusetts 01730

13. ABSTRACT

Reproduced by  
NATIONAL TECHNICAL  
INFORMATION SERVICE  
U S Department of Commerce  
Springfield VA 22151

A-28.5 (E)  
Cyt

8

AFCRC- TN - 59 - 768

UNITED STATES AIR FORCE  
CAMBRIDGE RESEARCH CENTER  
TECHNICAL LIBRARY  
DOCUMENT ROOM



AD751337

On Surface Electromagnetic Waves in Systems with Nonuniform  
Impedance

V. I. TALANOV

Izvestiia VUZ MVO, Radiofizika, vol. 2, No. 1, 1959, pp. 132 - 133.

MORRIS D. FRIEDMAN, INC.  
1383 A Washington Street  
West Newton 65, Massachusetts

T - 134

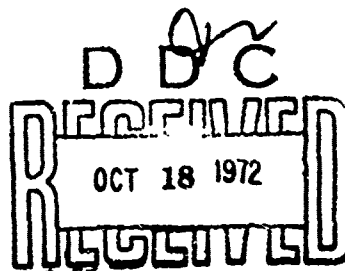
AF19(604)-5969 ✓

September 1959

Prepared  
for

ELECTRONICS RESEARCH DIRECTORATE  
AIR FORCE CAMBRIDGE RESEARCH CENTER  
AIR RESEARCH AND DEVELOPMENT COMMAND  
UNITED STATES AIR FORCE  
BEDFORD, MASSACHUSETTS

Approved for public release; distribution unlimited.



1a B -

R

On Surface Electromagnetic Waves in Systems with Nonuniform Impedance

V. I. TALANOV

Izvestia VUZ MVO, Radiofizika, vol. 2, No. 1, 1959, pp 132-133

is known, nonuniformities in the surface impedance [1] in unshielded retarding systems generally lead to the transformation of the surface wave energy into radiation field energy [2]. Moreover, there are cases when the system with a nonuniform impedance permits the existence of surface waves, which are analogous to surface waves in systems with uniform impedance with respect to both the field configuration and in that they are normal waves orthogonal to fields of other types.

As an example, let us analyze two-dimensional waves with the component  $H_z \neq 0$  ( $r, \varphi, z$  are cylindrical coordinates) within the two-sided angle ( $\varphi = 0, \varphi = \varphi_0, 0 < \varphi_0 \leq 2\pi$ ) on the faces of which are prescribed the homogeneous boundary conditions

$$E_r = Z^{(1)}(r)H_z|_{\varphi=0}; \quad E_r = -Z^{(2)}(r)H_z|_{\varphi=\varphi_0}$$

Let the surface impedances  $Z^{(1)}, Z^{(2)}$  depend on the  $r$  coordinate as  $\frac{1}{r}$ :

$$Z^{(1)} = i \frac{q_1}{kr} Z_0; \quad Z^{(2)} = i \frac{q_2}{kr} Z_0 \quad (\text{Im } q_1 = \text{Im } q_2 = 0)$$

( $k = \omega\sqrt{\mu\epsilon}$ ;  $Z_0 = \sqrt{\frac{\mu}{\epsilon}}$ ;  $\mu, \epsilon$  are the parameters of the homogeneous medium filling the space between the faces). The following functions will be solutions of the wave equation for  $H_z$

$$(1) \quad H_{z_m} = H_{\nu_m}^{(1,2)}(kr) [q_1 \sin \nu_m \varphi - \nu_m \cos \nu_m \varphi]$$

where  $H_{\nu_m}^{(1,2)}(kr)$  is the Hankel function of order  $\nu_m$  of the first and second kind and  $\nu_m$  are the roots of the characteristic equation

$$(2) \quad (q_1 q_2 - \nu_m^2) \tan \nu_m \varphi_0 = \nu_m (q_1 + q_2)$$

In addition to an infinite number of real roots, equation (2) has either one or two pure imaginary roots  $\nu_k = i\nu_k$  for definite values of the parameters  $q_1, q_2$  and the angle  $\varphi_0$ , which correspond to waves similar to slow waves between two parallel impedance planes.

In the particular case when  $q_1 = -q_2 = q > 0$ , along with the eigenfunctions (1) for  $\nu_m = \frac{m\pi}{\varphi_0}$  ( $m = 1, 2, 3, \dots$ ), the following functions will also

be solutions

$$(3) \quad H_{z_0} = H_{1q}^{(1,2)}(kr) e^{-q\psi}$$

These functions describe a field decreasing exponentially in azimuth  $\psi$  and independent of the angle  $\psi_0$ . The latter can be taken equal to  $\pi$ , say, which corresponds to the two-sided angle becoming a plane. It is not difficult to show that the functions  $H_{z_m}(r, \psi)$  ( $m = 1, 2, \dots$ ) and  $H_{z_0}(r, \psi)$  form an orthogonal system in the interval  $(0 \leq \psi \leq \psi_0)$ .

Under the condition that  $\exp(-q\psi_0) \ll 1$ , the face  $\psi = \psi_0$  plays almost no part in the formation of the wave (3). Hence, we arrive at a surface wave with a cylindrical front which is propagated along a plane with a nonuniform impedance.

Substituting the asymptotic expressions [3] in (3) for the Hankel functions with large arguments  $kr \gg q$ , we obtain the eigenfunctions

$$(4) \quad H_{z_0} \sim \sqrt{\frac{2Q(r)}{\pi q}} \sqrt{1 + Q^2(r)} \exp\left[\pm i \int k \sqrt{1 + Q^2(r)} dr - q\psi + C\right]$$

which describe surface waves with slowly varying amplitude and phase rate.  $C$  is a constant and  $Q(r) = \frac{q}{kr}$  in (4).

The system considered is the simplest model of a nonuniform, retarding structure which permits the picture of wave propagation to be described sufficiently graphically. Moreover, it is of known interest from the viewpoint of certain applications also. Thus, for example, the use in plane antennas of surface waves of retarding systems with a surface impedance varying as  $\frac{1}{r}$ , affords the possibility not only of obtaining the required dimensions of the effective antenna aperture but also (which is no less important) of making a computation of the antenna just as in the computation of horn emitters.

The method used above of separating variables in curvilinear coordinates (cylindrical in this case) can be used to solve problems of wave propagation for certain other dependences of the surface impedance on the coordinate, which are related in a definite way to the Lamé parameters for the appropriate coordinate system. It should here be kept in mind that nonuniform waves, localized to some degree or other at the impedance surfaces, cannot in the general case also have the character of surface fields such as (3) which decrease monotonically and rapidly enough (purely exponentially, say) upon removal from the

surface in the direction of the change of the appropriate curvilinear coordinate.

Radio Physics Research Inst. of Gor'kii Univ.

Dec. 17, 1958

References

1. M. A. MILLER: DAN USSR, 87, 571 (1952) (Translated: RT-1452)
2. G. WEILL: Annale Radioel., 10, 228 (1955)
3. G. N. WATSON: Bessel Functions.